Math 055 Exam 3
Summer 2025

For full credit: Please show work using techniques from this course and use correct mathematical notation.

1. (5 pts) Solve the system by finding eigenvalues and eigenvectors.

$$\frac{dx}{dt} = 4x - 3y$$
$$\frac{dy}{dt} = 3x + 4y$$

2. (4 pts) The eigenvalue for the system $\mathbf{X}' = \begin{pmatrix} 1 & -3 \\ 3 & 7 \end{pmatrix}\mathbf{X}$ is $\lambda = 4$ (multiplicity 2). Use this information to find the general solution to the system.

3. (3 pts) The eigenvalue for the system $\mathbf{X}' = \begin{pmatrix} 0 & 1 & 2 \\ -5 & -3 & -7 \\ 1 & 0 & 0 \end{pmatrix}\mathbf{X}$ is $\lambda = -1$ (mult. 3). Use this information and the information below to find the general solution to the system.

$\mathbf{K} = \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$, $\mathbf{P} = \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

4. (5 pts) The complementary function of a system is given. Use variation of parameters to find the particular solution of the system.

$$\mathbf{X} = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}\mathbf{X} + \begin{pmatrix} t^{-1} \\ 4 + 2t^{-1} \end{pmatrix}, \qquad \mathbf{X}_c(t) = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-5t}$$

5. (10 pts) Find two power series solutions of the differential equation about the ordinary point $x = 0$. Please include three terms for each solution if possible.
   $y'' - xy' + 4y = 0$

6. (6 pts) Consider the differential equation $2x^2y'' - xy' + (x^2 + 1)y = 0$. Use the method of Frobenius to find the indicial roots and recurrence relation (in terms of r and k) for the equation. (That is, find the values of r and the recurrence relation and then stop. Don't plug the r-values into the recurrence relation.)